Passive-Radiator Loudspeaker Systems Part I: Analysis*

RICHARD H. SMALL

School of Electrical Engineering, The University of Sydney, Sydney, N.S.W. 2006, Australia

The passive-radiator loudspeaker system is a close relative of the vented-box system and is capable of similar low-frequency performance. The passive radiator may be of any area but should preferably have a suspension with high compliance and low mechanical losses. It should also possess a linear volume displacement limit at least twice that of the system driver.

1. INTRODUCTION

Historical Background

The use of passive radiators in direct-radiator loudspeaker systems was described by Olson in a U.S. patent of 1935 [1]. Apparently, commercial exploitation of the principle was not immediate. The first description of the physical performance of such a loudspeaker system was published by Olson in 1954 [2]. Olson made direct comparisons between the use of a vent and a passive radiator (or drone cone) with the same driver and enclosure and claimed several advantages in favor of the passive radiator [2], [3].

Despite the very favorable results reported by Olson, only a few manufacturers have attempted to produce passive-radiator loudspeaker systems commercially. Perhaps an important reason for the limited interest in these systems has been the lack of any comprehensive published quantitative analysis or guide to their design.[1]

* Abridged version of this paper was presented September 10, 1973, at the 46th Convention of the Audio Engineering Society, New York

[1] This was written before the publication of the small-signal analysis by Nomura and Kitamura [9]. The present paper uses a slightly different approach, contains a somewhat wider range of useful alignments, and also deals with large-signal performance and design.

Technical Background

The passive-radiator loudspeaker system is a direct-radiator system using an enclosure which has two apertures. One aperture accommodates a driver, the other contains a suspended diaphragm which may resemble a driver but which has no voice coil or magnet assembly. The second undriven diaphragm is variously called a drone cone, passive radiator, or auxiliary bass radiator.

At low frequencies, the passive-radiator diaphragm moves in response to pressure variations within the enclosure [1]. It thus contributes to the total volume velocity crossing the enclosure boundaries and therefore to the system acoustic output [4].

The operation of the passive-radiator system is very similar to that of the vented-box system [5], the principal difference being the presence of a compliant suspension in the passive radiator which is not present with a simple vent. Because of this similarity, the passive-radiator system can be expected to perform in a manner similar to the vented-box system if passive-radiator compliance is made large enough.

In Part I of this paper, the passive-radiator system is analyzed by the general method described in [4]. Important objectives of this analysis are to determine the effects of limited passive-radiator compliance and to discover any advantages or disadvantages of this system compared to the vented-box system. The basic analytical results reveal the important physical relationships governing the small-signal and large-signal performance of passive-radiator systems and provide a quantitative basis for the measurement, assessment, and design of these systems.

Part II will provide a discussion of these results and present methods of synthesis (system design) which facilitate the design of an enclosure and passive radiator for a given driver or the specification of all system components required to meet a complete and realizable set of system performance specifications.

2. BASIC ANALYSIS

The impedance-type acoustical analogous circuit of a passive-radiator loudspeaker system is presented in Fig. 1.

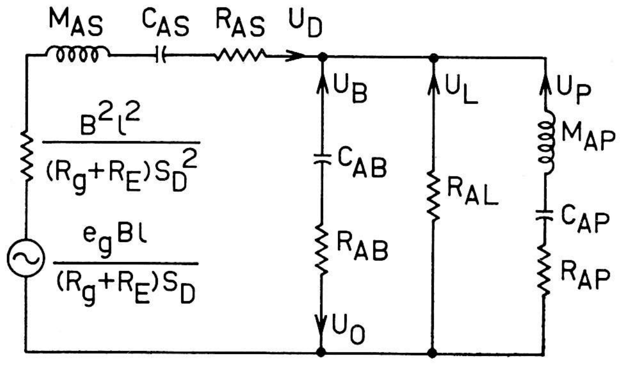


Fig. 1. Acoustical analogous circuit of passive-radiator loudspeaker system.

The symbols in this circuit are defined as follows.

e_g	open-circuit (Thevenin) output voltage of source or amplifier
B	magnetic flux density in driver air gap
l	length of voice-coil conductor in magnetic field of air gap
S_D	effective projected surface area of driver diaphragm
R_g	output (Thevenin) resistance of source or amplifier
R_E	dc resistance of driver voice coil
C_{AS}	acoustic compliance of driver suspension
M_{AS}	acoustic mass of driver diaphragm assembly including voice coil and air load
R_{AS}	acoustic resistance of driver suspension losses
C_{AB}	acoustic compliance of air in enclosure
R_{AB}	acoustic resistance of enclosure losses contributed by internal energy absorption
R_{AL}	acoustic resistance of enclosure losses contributed by leakage
C_{AP}	acoustic compliance of passive-radiator suspension
M_{AP}	acoustic mass of passive-radiator diaphragm including air load
R_{AP}	acoustic resistance of passive-radiator suspension losses
U_D	volume velocity of driver diaphragm
U_P	volume velocity of passive-radiator diaphragm
U_L	volume velocity of enclosure leakage
U_B	volume velocity entering enclosure
U_0	total volume velocity leaving enclosure boundaries.

This circuit may be simplified by combining the series resistances in the driver branch to form a single acoustic resistance R_{AT} where

$$R_{AT} = R_{AS} + \frac{B^2l^2}{(R_g + R_E)S_D{}^2} \tag{1}$$

by defining

$$p_g = \frac{e_g Bl}{(R_g + R_E)S_D} \tag{2}$$

as the value of the pressure generator at the left of the circuit, and by ignoring losses in the enclosure and passive radiator. The effects of these losses are examined indirectly later in the paper. The simplified circuit is presented in Fig. 2.

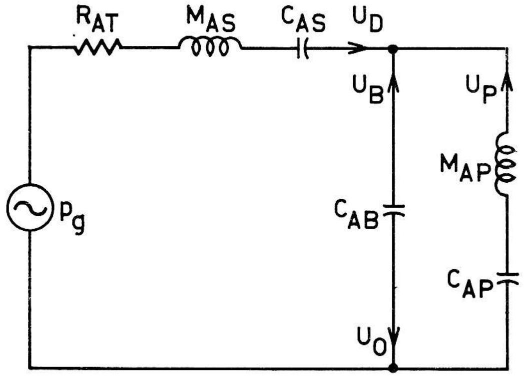


Fig. 2. Simplified acoustical analogous circuit of passive-radiator loudspeaker system with no enclosure or passive-radiator losses.

The complete electrical equivalent circuit of the passive-radiator system is the dual of Fig. 1. The electrical circuit elements are related to the acoustical circuit elements by the relationship

$$Z_E = \frac{B^2l^2}{S_D{}^2\ Z_A} \tag{3}$$

where Z_E is the impedance of an element in the electrical equivalent circuit and Z_A is the impedance of the corresponding element in the acoustical analogous circuit.

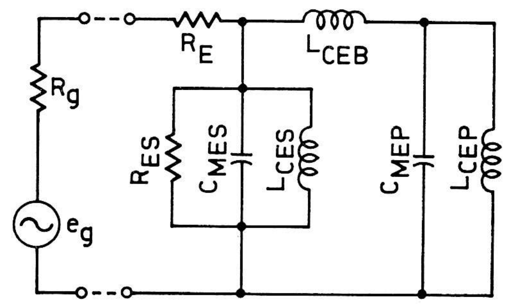


Fig. 3. Simplified electrical equivalent circuit of passive-radiator loudspeaker system.

A simplified electrical equivalent circuit corresponding to Fig. 2 is presented in Fig. 3. The symbols in this circuit are defined as follows.

C_{MES}	electrical capacitance representing driver mass, $= M_{AS}S_D{}^2/(Bl)^2$
L_{CES}	electrical inductance representing driver suspension compliance $= C_{AS}\ B^2l^2/S_D{}^2$
R_{ES}	electrical resistance representing driver suspension losses $= B^2l^2/(S_D{}^2R_{AS})$

L_{CEB} electrical inductance representing enclosure compliance, $= C_{AB}B^2l^2/S_D^2$

C_{MEP} electrical capacitance representing passive-radiator mass, $= M_{AP}S_D^2/(Bl)^2$

L_{CEP} electrical inductance representing passive-radiator suspension compliance, $= C_{AP}B^2l^2/S_D^2$.

The circuit of Fig. 3 has been arranged so that the actual system voice-coil terminals are accessible. This facilitates the study of the system voice-coil impedance and its relationship to the system element values.

The circuits presented above are valid only for frequencies within the piston range of the system driver. The element values are assumed to be independent of frequency within this range.

As discussed in [4], both voice-coil inductance and radiation load resistance are neglected in the construction of these circuits. Also neglected is the effect of external acoustic interaction between driver and passive radiator; this approximation is justified later in the paper.

The analysis of the system and the interpretation of its describing functions are simplified by defining a number of component and system parameters. For the driver, these are [4]

$$T_S^2 = 1/\omega_S^2 = C_{AS}M_{AS} = C_{MES}L_{CES} \tag{4}$$

$$Q_{MS} = \omega_S C_{MES}R_{ES} = 1/(\omega_S C_{AS}R_{AS}) \tag{5}$$

$$Q_{ES} = \omega_S C_{MES}R_E = \omega_S R_E M_{AS} S_D^2/(Bl)^2 \tag{6}$$

$$V_{AS} = \rho_0 c^2 C_{AS} \tag{7}$$

Eq. (4) defines the resonance frequency of the driver ($\omega_S = 2\pi f_S$). In Eq. (7) ρ_0 is the density of air (1.18 kg/m^3) and c is the velocity of sound in air (345 m/s). Eq. (7) expresses the acoustic compliance of the driver suspension in terms of a volume of air (under standard conditions of temperature and pressure) which has the same acoustic compliance. In this paper it is assumed that M_{AS} and hence the values of f_S, Q_{MS}, and Q_{ES} apply to the driver when the diaphragm air-load mass has the value normally imposed by the system enclosure; where appropriate, this is indicated explicitly by using the symbol f_{SB} for f_S [4], [5].

Similar parameters are defined for the passive radiator, except that there is no equivalent to Q_{ES}. There is only one Q, related to suspension losses. Thus,

$$T_P^2 = 1/\omega_P^2 = C_{AP}M_{AP} = C_{MEP}L_{CEP} \tag{8}$$

$$Q_{MP} = \omega_P C_{MEP}R_{EP} = 1/(\omega_P C_{AP}R_{AP}) \tag{9}$$

$$V_{AP} = \rho_0 c^2 C_{AP}. \tag{10}$$

It is assumed in this paper that the values of ω_P (or the corresponding f_P) and Q_{MP} apply to the passive radiator when the diaphragm air-load mass has the value normally imposed by the system enclosure.

The enclosure, with the passive radiator installed, exhibits a resonance frequency $\omega_B = 2\pi f_B$ in the same manner as does a vented enclosure. This frequency is given by

$$T_B^2 = \omega_B^2 l = \frac{C_{AB}M_{AP}}{1+\dfrac{C_{AB}}{C_{AP}}} = \frac{C_{MEP}L_{CEB}}{1+\dfrac{L_{CEB}}{L_{CEP}}}. \tag{11}$$

The losses in the enclosure and passive radiator are conveniently defined as Q at the enclosure resonance frequency in the same manner as for the vented-box system [5, sec. 3]. Thus for absorption, leakage, and passive-radiator suspension losses respectively,

$$Q_A = 1/(\omega_B C_{AB}R_{AB}) \tag{12}$$

$$Q_L = \omega_B C_{AB}R_{AL} \tag{13}$$

$$Q_P = 1/(\omega_B C_{AB}R_{AP}). \tag{14}$$

The total enclosure loss Q_B at f_B is then given by

$$1/Q_B = 1/Q_A + 1/Q_L + 1/Q_P. \tag{15}$$

The interaction of the source, driver, enclosure, and passive radiator give rise to further system parameters. These are the system compliance ratio

$$\alpha = C_{AS}/C_{AB} = L_{CES}/L_{CEB} \tag{16}$$

the passive-radiator compliance ratio

$$\delta = C_{AP}/C_{AB} = L_{CEP}/L_{CEB} \tag{17}$$

the system tuning ratio

$$h = f_B/f_S = \omega_B/\omega_S = T_S/T_B \tag{18}$$

the passive-radiator tuning ratio

$$y = f_P/f_S = \omega_P/\omega_S = T_S/T_P \tag{19}$$

and the total Q of the driver connected to the source

$$Q_T = 1/(\omega_S C_{AS}R_{AT}). \tag{20}$$

In dealing with the system-describing functions it is useful to recognize that from Eqs. (8), (11), and (17)–(19)

$$T_P/T_B = f_B/f_P = h/y = (\delta+1)^{1/2}. \tag{21}$$

Following the method of [4], analysis of Figs. 2 and 3, and substitution of the parameters defined above yields the system-describing functions. The response function is

$$G(s) = \frac{s^2T_S^2(s^2T_P^2+1)}{D(s)} \tag{22a}$$

where

$$\begin{aligned} D(s) = {} & s^4T_P^2T_S^2 + s^3T_P^2T_S/Q_T \\ & + s^2[(\alpha+1)T_P^2 + (\delta+1)T_S^2] \\ & + s(\delta+1)T_S/Q_T + (\alpha+\delta+1) \end{aligned} \tag{22b}$$

and $s = \sigma+j\omega$ is the complex frequency variable.

The displacement function for the driver diaphragm, normalized to unity at zero frequency, is

$$X(s) = \frac{(\alpha+\delta+1)\ (s^2T_B^2+1)}{D(s)} \tag{23}$$

and the displacement constant is

$$k_x = \frac{\delta+1}{\alpha+\delta+1}. \tag{24}$$

Because the displacement capability of a passive-radiator diaphragm is limited by the suspension design, it is important to assess the required displacement as a function of frequency and power level. It is easily shown that at zero frequency the volume displacement of the passive radiator is equal to that of the driver multiplied by the factor $\delta/(\delta+1)$. The displacement function for the passive-radiator diaphragm $X_P(s)$, normalized to unity at zero frequency, is then given by

$$X_P(s) = \frac{(\alpha+\delta+1)}{D(s)} . \quad (25)$$

Analysis of the electrical equivalent circuit of Fig. 3 for the impedance of the circuit to the right of the voice-coil terminals gives the system voice-coil impedance function

$$Z_{VC}(s) = R_E + R_{ES}\frac{(\delta+1)\ (sT_S/Q_{MS})\,(s^2T_P^2 + 1)}{D'(s)} \quad (26)$$

where $D'(s)$ is the function $D(s)$ of Eq. (22) but with Q_T wherever it appears replaced by Q_{MS}.

3. RESPONSE

Response Function

The response function of the passive-radiator system given by Eq. (22) may be rearranged into the general form

$$G(s) = \frac{s^4T_0^4 + b_2s^2T_0^2}{s^4T_0^4 + a_1s^3T_0^3 + a_2s^2T_0^2 + a_3sT_0 + 1} . \quad (27)$$

This response function has a fourth-order denominator polynomial which is similar to that of the vented-box system. But unlike the vented-box system, two of the zeros of the numerator are located away from the origin of the s plane. It is the relocation of these zeros, caused by the passive-radiator suspension compliance, which makes the response of a passive-radiator system different from that of a comparable vented-box system.

Frequency Response

The frequency response $|G(j\omega)|$ of Eq. (27) is examined in the Appendix; coefficient values are given for a variety of system alignments which have useful response characteristics.

The distinguishing feature of the frequency response of the passive-radiator system is the presence of a notch or dip which appears at the resonance frequency f_P of the passive radiator as indicated by Eq. (22a). This frequency is normally located below the system cutoff frequency. The effect of the notch generally is to sharpen the "corner" of the frequency response characteristic and to give a steeper initial cutoff slope compared to the equivalent vented-box system.

In this respect the passive-radiator system response may be loosely compared to that of the "m-derived" high-pass filter of classical image-parameter theory and the vented-box system response to that of the "constant-k" high-pass filter [6, pp. 181-183, 652]. In the terminology of the modern insertion-loss filter theory on which the Appendix is based, the passive-radiator system response is that of an elliptic-function filter [7, pp. 489, 532].

Alignment

The response notch of the passive-radiator system may be eliminated by adjusting the system parameters so that two of the denominator poles exactly cancel the numerator zeros contributing the notch. Considerable damping must be introduced into the passive radiator to achieve this. The result is a system with pure second-order response (a nominal 12-dB per octave cutaff slope), but unfortunately one which is demonstrably inferior to a normal closed-box system in terms of the efficiency constant and power rating constant obtained [8].

Allowing the notch to remain, the high-pass behavior of the system above the notch frequency can be made to have equal-ripple, maximally flat, or quasi maximally flat properties as discussed in the Appendix. The response characteristic below the notch frequency is not of particular interest because it is very far down in the stop band.

Comparison of Eqs. (22) and (27) reveals that the five mathematical variables required to specify a given alignment (T_0, a_1, a_2, a_3, and b_2) are related to the five independent system parameters (T_S, T_P, Q_T, α, and δ). This means that every specification of a particular alignment corresponds to a unique set of system parameters. However, unlike the simpler case of the vented-box system, specified *conditions* such as "maximally flat" do not define a unique set of coefficients for Eq. (27). There are now an infinite variety of maximally flat (passband) responses having notches at various frequencies below cutoff. Thus one system parameter may be specified arbitrarily if desired without necessarily restricting the range of types of passband alignments available; only the specific shape of each alignment type is fixed.

Fig. 4 illustrates some of the maximally flat responses[2] which may be obtained for various chosen values of the passive-radiator compliance ratio δ. As the value of δ approaches infinity (infinite passive-radiator compliance, and hence f_P or notch frequency of zero), the response characteristic approaches that of the pure fourth-order Butterworth alignment obtainable from the vented-box system [5].

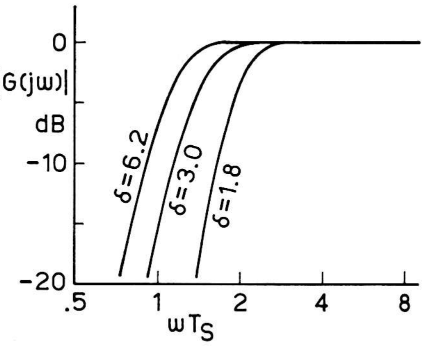


Fig. 4. Maximally flat passband responses obtainable from the passive-radiator loudspeaker system.

Fig. 5 is an alignment chart based on the range of maximally flat alignments obtainable from the lossless passive-radiator system, including those illustrated in Fig. 4. The system compliance ratio α is chosen as the primary independent variable and plotted as the abscissa. The curves then give the values of h (or y), δ, and Q_T required to obtain a maximally flat alignment as well as the normalized half-power cutoff frequency f_3/f_S at which the response is 3 dB below the passband reference level. Note that for the lossless passive-radiator system, maximally flat responses can be obtained only for values of α

[2] The maximally flat alignments of this paper are identical with the general Butterworth alignments of [9].

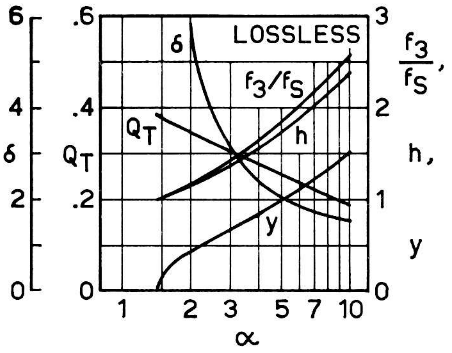


Fig. 5. Alignment chart for lossless passive-radiator system providing maximally flat passband responses.

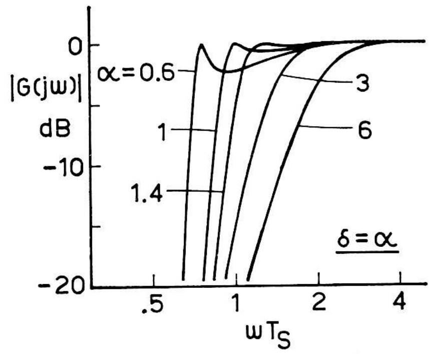


Fig. 6. Responses obtainable from passive-radiator system for the condition $\delta=\alpha$ (equal passive-radiator and driver compliances).

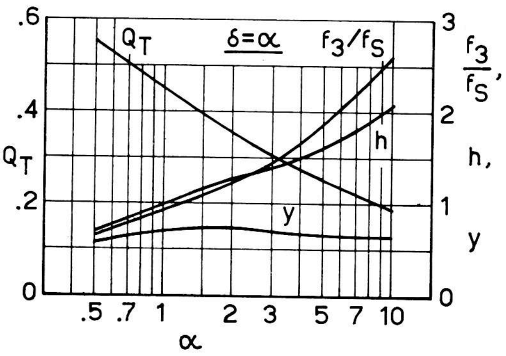


Fig. 7. Alignment chart for lossless passive-radiator systems with $\delta=\alpha$.

that are equal to or larger than the value required ($\sqrt{2}$) for a lossless vented-box system.

It may be shown from Eq. (22) that if the passive-radiator compliance is made infinite, the response is the same as for the vented-box system i.e., Eq. (22) reduces to [5, eq. (13)]. However, a common practical condition in a passive-radiator system is $\delta=\alpha$. This is because the passive radiator is often made from the same frame and suspension as the driver; the diaphragm is simply made heavier and the magnet and voice coil omitted. For the condition $\delta=\alpha$, Fig. 6 illustrates some of the response characteristics obtainable from the passive-radiator system. These include equal-ripple, maximally flat, and quasi maximally flat alignments.[3]

[3] The equal-ripple alignments used in this paper have negative ripple and are not the same kind used in [9]; those alignments have positive ripple and are obtained for somewhat different conditions. Both kinds are useful but possess slightly different values of the efficiency factor $k_{\eta(G)}$.

Fig. 7 is an alignment chart for lossless passive-radiator systems with $\delta=\alpha$. The range of alignments include those illustrated in Fig. 6. For a value of α very close to 3, the response is maximally flat. For lower values of α, the response is equal-ripple; for higher values of α, the response is quasi maximally flat.

Misalignment

The effect of an incorrectly adjusted parameter on the frequency response of a passive-radiator system is illustrated in Figs. 8 and 9. These curves were obtained with the use of an analog simulator. Fig. 8 shows the variation produced in the response of the lossless $\delta=\alpha$ maximally flat alignment by changes in the value of Q_T of $\pm 20\%$, -50%, and $+100\%$ from the nominally correct value. Fig. 9 shows the variations produced in the response of the same alignment by mistuning (a change in value of h or f_B) of $\pm 20\%$ and $\pm 50\%$. The effects are very similar to those for the vented-box system [5, Figs. 7 and 8], as might be expected.

System Losses

It can be expected in practice that Q_A and Q_L will have about the same values for a passive-radiator system as for a comparable vented-box system, provided that no additional leakage is introduced by such sources as faulty passive-radiator sealing gaskets. However, Q_P may be ex-

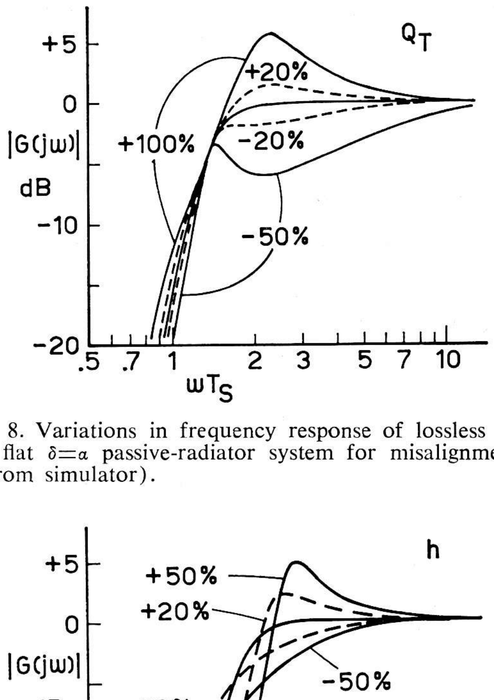


Fig. 8. Variations in frequency response of lossless maximally flat $\delta=\alpha$ passive-radiator system for misalignment of Q_T (from simulator).

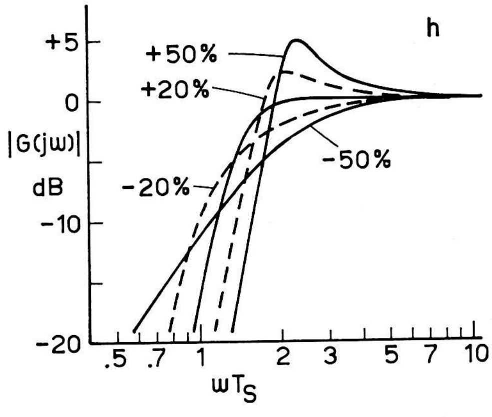


Fig. 9. Variations in frequency response of lossless maximally flat $\delta=\alpha$ passive-radiator system for misalignment of h (from simulator).

pected to be lower for the passive-radiator system, because R_{AP} in this system is commonly of the same order of magnitude as R_{AS}.

The effects of enclosure losses in the passive-radiator system can be evaluated by introducing finite values of Q_A, Q_L, and Q_P into a correctly aligned lossless system. Fig. 10 shows the effect of Q values of 5 on the lossless $\delta = \alpha$ maximally flat alignment, obtained by analog simulation. Fortunately, passive-radiator losses have the least effect on the system response.

All this suggests that the passive-radiator system will

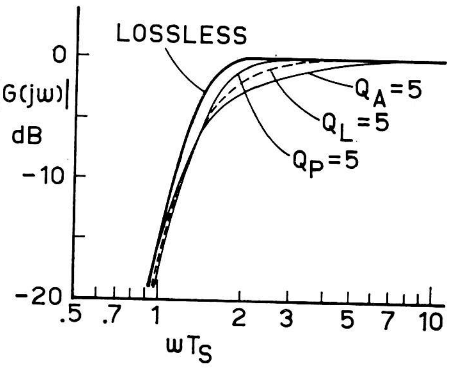


Fig. 10. Effects of enclosure and passive-radiator losses on response of a lossless maximally flat $\delta = \alpha$ passive-radiator system (from simulator).

exhibit a lower measured value of Q_B than its vented-box counterpart, but that the total effect of this loss on response will be only slightly greater. The lower value of Q_B has been confirmed by measurement on a number of passive-radiator systems for which the passive radiator could be replaced by an adaptor plate and a vent giving the same value of f_B.

Alignment with Enclosure Losses

The exact alignment parameters for lossy passive-radiator systems are extremely difficult to calculate from the relevant expanded form of Eq. (22). For this investigation, a shortcut was taken by observing the effects of losses on the vented-box system alignment and modifying the lossless passive-radiator system alignments similarly. The resulting alignments were tested by analog simulation and corrected as necessary to produce the desired response shapes. The final alignment data were then used

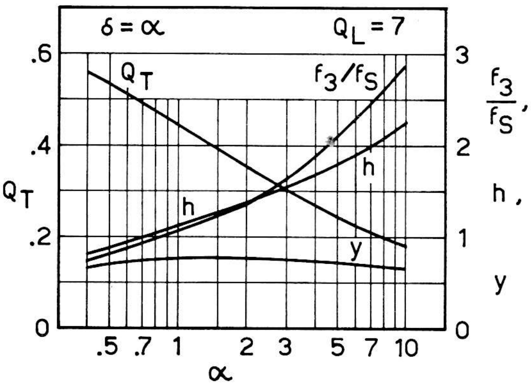


Fig. 11. Alignment chart for $\delta = \alpha$ passive-radiator systems with $Q_B = Q_L = 7$.

to construct the alignment chart of Fig. 11. This covers the same $\delta = \alpha$ alignments as Fig. 7, but for the condition $Q_B = Q_L = 7$. This condition is so typical of the total-loss structure of a wide variety of passive-radiator systems that have been tested (actual measured Q_B of 5) that no alignment charts for other values would appear to be useful. As a representation of typical conditions, Fig. 11 may be compared directly with [5, Fig. 11] for vented-box systems with $Q_B = Q_L = 7$.

Transient Response

The step responses of a selection of $\delta = \alpha$ lossless passive-radiator alignments are presented in Fig. 12. If these

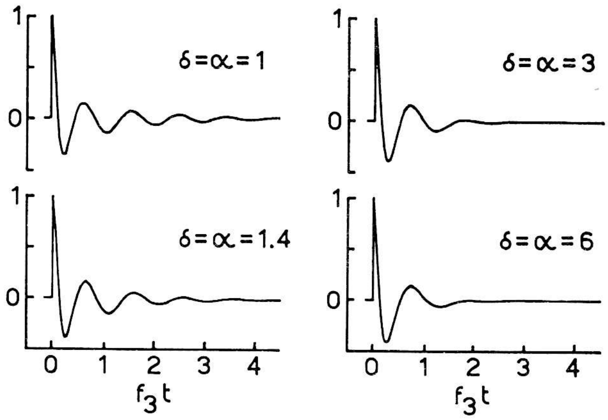


Fig. 12. Normalized step response of passive-radiator loudspeaker system (from simulator).

are compared to the corresponding step responses of equivalent vented-box alignments [5, Fig. 14], it is clear that the steeper cutoff slopes of the passive-radiator system contribute greater overshoot and transient ringing, particularly for systems with low compliance ratios. However, as pointed out earlier, it is the value of δ which is of greatest importance. If δ is made high, then even the low-α alignments for the passive-radiator system become very much like their vented-box system counterparts.

4. EFFICIENCY

Reference Efficiency

The piston-range reference efficiency η_0 of the passive-radiator system is the reference efficiency of the system driver when the total air-load mass on the driver diaphragm is that imposed by the enclosure. Thus [4, eq. (32)],

$$\eta_0 = \frac{4\pi^2}{c^3} \cdot \frac{f_S{}^3 V_{AS}}{Q_{ES}}. \tag{28}$$

Efficiency Factors

Following the method of [5, sec. 5], Eq. (28) may be put into the form

$$\eta_0 = k_\eta \, f_3{}^3 \, V_B \tag{29}$$

where f_3 is the half-power or -3-dB cutoff frequency of the system, V_B is the net internal volume of the system enclosure, and k_η is an efficiency constant consisting of two factors; namely,

$$k_\eta = k_{\eta(Q)} k_{\eta(G)} \tag{30}$$

where

$$k_{\eta(Q)} = Q_T/Q_{ES} \tag{31}$$

$$k_{\eta(G)} = \frac{4\pi^2}{c^3} \cdot \frac{V_{AS}}{V_B} \cdot \frac{f_S^3}{f_3^3} \cdot \frac{1}{Q_T} . \tag{32}$$

Driver Loss Factor

If $R_g = 0$, then $Q_T = Q_{TS}$, where

$$Q_{TS} = \frac{Q_{ES}Q_{MS}}{Q_{ES}+Q_{MS}} . \tag{33}$$

Thus

$$k_{\eta(Q)} = Q_{TS}/Q_{ES} = 1 - \frac{Q_{TS}}{Q_{MS}} . \tag{34}$$

This efficiency factor reflects the effects of mechanical losses in the system driver. For typical drivers used in passive-radiator systems, $k_{\eta(Q)}$ has a value in the range of 0.8 to 0.95.

System Response Factor

For normal passive-radiator system enclosures containing only a small amount of damping material used as a lining,

$$C_{AB} = V_B/(\rho_0 c^2) \tag{35}$$

and Eq. (32) can be written as

$$k_{\eta(G)} = \frac{4\pi^2}{c^3} \cdot \frac{\alpha}{Q_T(f_3/f_S)^3} . \tag{36}$$

For any passive-radiator system alignment contained in Figs. 7 or 11, the values of α, Q_T, and f_3/f_S are known

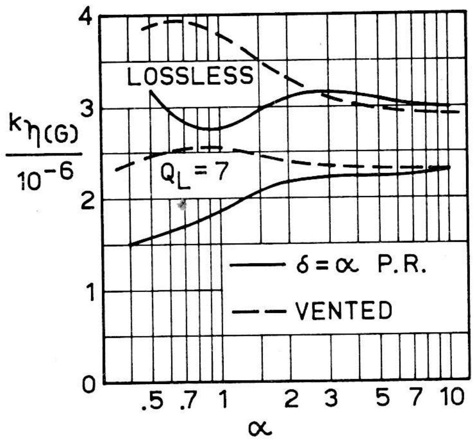


Fig. 13. Response factor $k_{\eta(G)}$ of efficiency constant for $\delta=\alpha$ passive-radiator systems (solid lines) and vented-box systems (broken lines) with lossless enclosures and with $Q_B = Q_L = 7$.

and the value of $k_{\eta(G)}$ may be calculated. Fig. 13 is a plot of the value of $k_{\eta(G)}$ as a function of α for Q_L equal to 7 and infinity. For comparison, the corresponding curves for vented-box systems [5, Fig. 15] are shown by broken lines. Note that the pairs of curves differ only in the value of δ; this is infinite for the vented-box system but equal to α for the passive-radiator system. Thus for the alignment types included here, there is little difference in $k_{\eta(G)}$ for δ values above about 2; lower values, however, place the passive-radiator system at a definite disadvantage.

5. DISPLACEMENT-LIMITED POWER RATINGS

Driver Diaphragm Displacement

The passive-radiator system displacement function given by Eq. (23) has essentially the same form as that for the vented-box system [5, eq. (14)]. However, k_x for the passive-radiator system, as given by Eq. (24), is less than unity. This indicates that for very low frequencies at least, the driver diaphragm displacement for the passive-radiator system is less than that for the vented-box system. Fig. 14 is a plot of $k_x|X(j\omega)|$ for several of the lossless $\delta=\alpha$ passive-radiator system alignments. The frequency scale is normalized to f_B. As expected, this plot

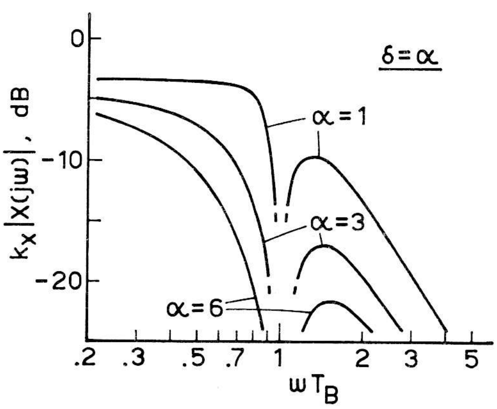


Fig. 14. Normalized diaphragm displacement of passive-radiator system driver as a function of normalized frequency for several typical $\delta=\alpha$ lossless alignments (from simulator).

is very similar to the corresponding vented-box data [5, Fig. 17], except at very low frequencies. But the low-frequency displacement decrease is not large.

From Eq. (24) the displacement at very low frequencies can be reduced by up to 6 dB if $\delta=\alpha>>1$. Significantly greater reduction is possible only if α is large and δ is small. Because small values of δ lead to rather poor performance in terms of transient response and the value of $k_{\eta(G)}$, it is clear that no dramatic reduction of very-low-frequency diaphragm displacement sensitivity over that of the vented-box system can be achieved with the passive-radiator system, unless a considerable sacrifice of performance can be tolerated.

Acoustic Power Rating

Assuming linear large-signal diaphragm displacement, the steady-state displacement-limited acoustic power rating P_{AR} of a loudspeaker system, from [4, eq. (42)], is

$$P_{AR} = \frac{4\pi^3\rho_0}{c} \cdot \frac{f_S^4 V_D^2}{k_x^2|X(j\omega)|_{max}^2} \tag{37}$$

where $|X(j\omega)|_{max}$ is the maximum magnitude attained by the displacement function and V_D is the peak displacement volume of the driver diaphragm. The latter is given by

$$V_D = S_D\, x_{max} \tag{38}$$

where x_{max} is the peak linear displacement of the driver diaphragm.

Eq. (37) may be written in the form

$$P_{AR} = k_P f_3^4 V_D^2 \tag{39}$$

where k_P is a power rating constant given by

$$k_P = \frac{4\pi^3\rho_0}{c} \cdot \frac{1}{(f_3/f_S)^4 \ (k_x|X(j\omega)|_{max}^2}\ . \tag{40}$$

Values of (f_3/f_S) may be calculated for any alignment. From Fig. 14 the quantity $k_x|X(j\omega)|$ has two maxima, one within and one below the passband, just as for the vented-box system. For the passband maxima, the magnitudes are very little different from those of comparable vented-box alignments. The alignment data are also similar, particularly for large δ. Thus for average program material having most of its energy within the system passband, the power ratings must be about the same as for vented-box systems, i.e. [5, eq. (41)],

$$P_{AR} = 3.0\, f_3^4\, V_D^2\ . \tag{41}$$

For a graphical illustration of this relationship between acoustic power rating, cutoff frequency, and driver displacement volume, see [5, Fig. 19].

Note that this rating is not affected by the displacement reduction that occurs at *very* low frequencies for the passive-radiator system, because this reduction does not extend to the frequency range near cutoff. However, it is reasonable to expect that the passive-radiator system should be somewhat less vulnerable to *very*-low-frequency signals such as amplifier turn-on and turn-off transients and the too hastily lowered pickup stylus.

Electrical Power Rating

The displacement-limited electrical input power rating P_{ER} of the passive-radiator system may be obtained by dividing the acoustic power rating by the system reference efficiency. The dependence of this rating on the important system parameters is observed by dividing Eq. (39) by Eq. (29):

$$P_{ER} = \frac{P_{AR}}{\eta_0} = \frac{k_P}{k_\eta} f_3 \frac{V_D^2}{V_B}\ . \tag{42}$$

6. PASSIVE-RADIATOR REQUIREMENTS

The effective surface area of the passive radiator is usually made equal to that of the driver. This condition is not necessary for successful operation, but several factors encourage it. It was stated earlier that the passive radiator is often made from the same frame and suspension as the driver; the economic advantages of this approach are readily apparent, and it results in equal areas.

The use of a passive radiator which is substantially larger than the driver is seldom feasible because of the required baffle area. In most cases the size of both driver and passive radiator are limited by the enclosure dimensions, and it is impractical to make the passive radiator area more than about twice that of the driver.

The alternative of making the passive radiator smaller than the driver is almost never encountered. The principal reason for this is that the volume displacement required of the passive radiator is quite substantial. A small area therefore requires a very large linear displacement capability which can be difficult to achieve in practice.

In Section 5 the power capacity of the passive-radiator system is determined on the basis that the limiting factor is the displacement volume of the driver. If this power capacity is to be realized in practice, the passive radiator must be designed so that it is capable of displacing the maximum volume required of it by the system at rated power output. This volume displacement requirement is normally larger than that of the driver and is the physical reason for the relatively high power rating constant of the system.

The relative maximum volume displacement requirements for the driver and passive radiator may be found from Eqs. (23) and (25), recognizing that at zero frequency the passive-radiator volume displacement must be $\delta/(\delta+1)$ of that of the driver as noted in Section 2. Fig. 15 illustrates the relative displacements as a function of

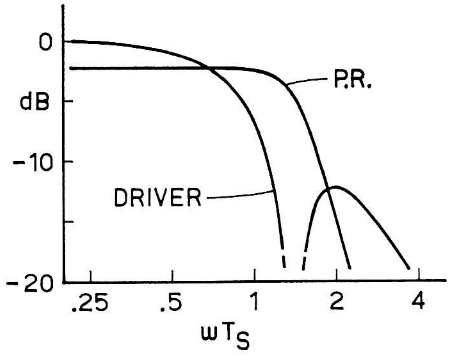


Fig. 15. Normalized displacements of driver and passive-radiator as a function of normalized frequency for lossless maximally flat $\delta=\alpha$ passive-radiator system alignment.

frequency for the lossless maximally flat $\delta=\alpha$ alignment. The maxima occur at different frequencies, but, most importantly, high passive-radiator displacement is required within the system passband.

For program-rated systems, the passive radiator displacement volume V_{PR} must typically be about twice the rated driver displacement volume V_D. Fig. 16 is a plot of the required ratio of V_{PR} to V_D as a function of α for all of the $\delta=\alpha$ alignments. If driver and passive radiator have the same effective surface areas, the maximum linear displacements must be in this ratio.

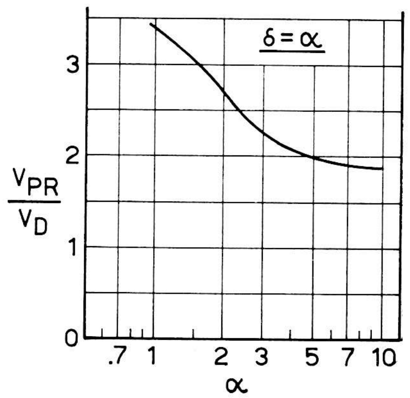


Fig. 16. Required ratio of passive-radiator displacement volume V_{PR} to driver displacement volume V_D as a function of α for program-rated $\delta=\alpha$ passive-radiator systems (from simulator).

Not all high-quality drivers have a suspension capable of more than twice the linear displacement that the magnet/voice-coil structure can provide with good linearity. For this reason, optimum design of passive-radiator systems may require that the passive-radiator suspension be somewhat different from that of the driver. The "convenience" of using the same suspension may in fact result in limited power capacity compared to that which could be achieved with a specially designed passive radiator.

An interesting feature of the $\delta=\alpha$ alignments is the small variation of the required value of $y=(f_P/f_S)$. For the most common alignments, a passive radiator made from the same frame and suspension as the driver (assuming adequate displacement capability) consistently requires a diaphragm mass almost twice that of the driver for correct system alignment.

The general requirements for a passive radiator may be summarized as acoustic mass and displacement volume roughly twice those of the driver, acoustic compliance equal to or greater than that of the driver, and suspension losses as low as possible.

7. MUTUAL COUPLING IN PASSIVE-RADIATOR SYSTEMS

Mutual coupling in passive-radiator systems takes the same form as for vented-box systems [5, sec. 8]. However, the effects are generally even smaller than for the vented-box system.

If the diameter of the passive radiator is equal to that of the driver, as is usual, the minimum center-to-center aperture spacing is greater than for the vented-box system, and the mutual coupling mass is therefore smaller. Furthermore, passive radiators are most often used in smaller loudspeaker systems which require relatively heavy driver cones to obtain extended low-frequency response. The mutual-coupling mass under these conditions represents only a tiny fraction of the total driver moving mass, giving quite negligible effects on both performance and measurement.

8. PARAMETER MEASUREMENT

Voice-Coil Impedance

The voice-coil impedance function of the passive-radiator system is given by Eq. (26). The steady-state magnitude $|Z_{VC}(j\omega)|$ of this function has the shape plot-

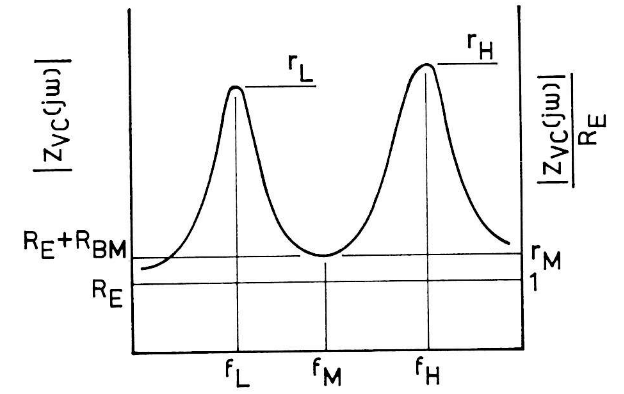


Fig. 17. Voice-coil impedance magnitude of passive-radiator loudspeaker system as a function of frequency.

ted in Fig. 17. This shape is exactly the same as that for the vented-box system [5, Fig. 20]. The plot has two maxima, at the frequencies labeled f_L and f_H. Between these maxima, there is a minimum at a frequency near f_B which is labeled f_M. At f_M the minimum impedance is slightly greater than R_E; the additional resistance is contributed by enclosure and passive-radiator losses and designated R_{BM}.

Small-Signal Parameter Measurement

The measured impedance curve of a passive-radiator system conforms closely to the shape of Fig. 17. The impedance maximum at f_L is usually lower than that at f_H because of passive-radiator losses. As in the case of the vented-box system, the basic system parameters may be evaluated with satisfactory accuracy by ignoring enclosure and passive-radiator losses for initial calculations and then calculating the system losses using the approximate system data.

Ignoring enclosure and passive-radiator losses, and assuming that $f_M \doteq f_B$, Eq. (26) may be used to derive the following parameter-impedance-plot relationships:

$$\frac{\delta+1}{\alpha+\delta+1}=\frac{f_B^2\, f_{SB}^2}{f_L^2\, f_H^2} \tag{43}$$

$$\frac{\alpha\,\delta}{\alpha+\delta+1}=\frac{(f_H+f_B)\,(f_H-f_B)\,(f_B+f_L)\,(f_B-f_L)}{f_L^2\, f_H^2}. \tag{44}$$

These relationships do not give an immediate solution for any of the passive-radiator system parameters as do their counterparts for the vented-box system [5, eqs. (44) and (45)]. This is because only the same amount of information is available from the impedance curve while the system has the additional parameter δ to be evaluated.

However, it is relatively easy to evaluate α. If the passive radiator can be removed from the enclosure, it can be replaced temporarily by a vent. Then f_{SB} and α can be calculated as for a vented-box system from [5, eqs. (44) and (45)]. The passive-radiator aperture can also be blocked off and α evaluated as for a closed-box system from [8, eq. (48)]. Alternatively, the driver resonance frequency f_S may be measured and adjusted to correspond to the air-load mass applicable in the enclosure; then, using the passive-radiator system impedance-plot data,

$$\alpha=\frac{f_H^2+f_L^2-f_B^2}{f_{SB}^2}-1 \tag{45}$$

where Eq. (45) is derived directly from Eqs. (43) and (44).

With α and f_{SB} determined, δ may be found from either Eq. (43) or Eq. (44). A useful check for errors of measurement, calculation, or approximation is the computation of δ from both equations and comparison of the values obtained. Using the measured values of δ and f_B, f_P may be calculated from Eq. (21).

The remaining system parameters are measured in the manner described in [4, Appendix] and [5, sec. 6]. The value of Q_B computed from [5, eq. (49)] includes the effect of passive-radiator losses; assigning a value about 30–40% greater than this to Q_L gives a very satisfactory picture of the system response for evaluation purposes.

REFERENCES

[1] H. F. Olson, "Loud Speaker and Method of Propagating Sound," U.S. Patent No. 1,988,250, application Feb. 17, 1934; patented Jan. 15, 1935.

[2] H. F. Olson, J. Preston, and E. G. May, "Recent Developments in Direct-Radiator High-Fidelity Loudspeakers," *J. Audio Eng. Soc.*, vol. 2, pp. 219-227 (Oct. 1954).

[3] H. F. Olson, *Acoustical Engineering* (Van Nostrand, Princeton, N. J., 1957) pp. 161-162.

[4] R. H. Small, "Direct-Radiator Loudspeaker System Analysis," *IEEE Trans. Audio Electroacoust.*, vol. AU-19, pp. 269-281 (Dec. 1971); republished in *J. Audio Eng. Soc.*, vol. 20, pp. 383-395 (June 1972).

[5] R. H. Small, "Vented-Box Loudspeaker Systems," *J. Audio Eng. Soc.*, vol. 21, pp. 363-372, 438-444, 549-554, 635-639 (1973).

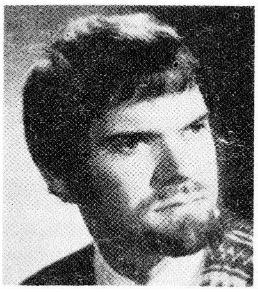
[6] F. Langford-Smith, *Radiotron Designer's Handbook*, 4th ed. (Wireless Press, Sydney, 1953).

[7] L. Weinberg, *Network Analysis and Synthesis* (McGraw-Hill, New York, 1962), ch. 11.

[8] R. H. Small, "Closed-Box Loudspeaker Systems," *J. Audio Eng. Soc.*, vol. 20, pp. 798-808 (Dec. 1972); vol. 21, pp. 11-18 (Jan./Feb. 1973).

[9] Y. Nomura and Z. Kitamura, "An Analysis of Design Conditions for a Phase-Inverter Speaker System with a Drone Cone," *IEEE Trans. Audio Electroacoust*, vol. AU-21, pp. 397-407 (Oct. 1973).

THE AUTHOR



Richard H. Small received the degrees of Bachelor of Science (1956) from the California Institute of Technology and Master of Science in Electrical Engineering (1958) from the Massachusetts Institute of Technology.

He was employed in electronic circuit design for high-performance analytical instruments at the Bell & Howell Research Center from 1958 to 1964, except for a one-year visiting fellowship to the Norwegian Technical University in 1962. After a working visit to Japan in 1964, he moved to Australia. In 1972, following the completion of a program of research into direct-radiator electrodynamic loudspeaker systems, he was awarded the degree of Doctor of Philosophy by The University of Sydney. He is now a Lecturer in Electrical Engineering at The University of Sydney.

Dr. Small is a member of the Institute of Electrical and Electronics Engineers, a member of the Institution of Radio and Electronics Engineers Australia, and a Fellow of the Audio Engineering Society.

Passive-Radiator Loudspeaker Systems Part II: Synthesis*

RICHARD H. SMALL

School of Electrical Engineering,
The University of Sydney,
N.S.W. 2006, Australia

Passive-radiator loudspeaker systems can be designed to specification as easily as vented-box systems. Driver requirements are generally about the same as for comparable vented-box systems, and the requirements of the passive radiator are directly related to those of the driver. The passive-radiator principle is particularly useful in compact systems where vent realization is difficult or impossible, but it can also be applied satisfactorily to larger systems.

INTRODUCTION: The analysis presented in Part I shows that the passive-radiator system is a very close relative of the vented-box system. The principal difference in performance is the presence of a notch in the frequency response below cutoff. While this notch can noticeably degrade performance, it can through the provision of high passive-radiator suspension compliance be placed so low in frequency that the system performance is virtually indistinguishable from that of a vented-box system in most fundamental respects.

However, the passive-radiator system has the distinct advantage that it is physically realizable in many cases where the vented-box system is not. This is particularly true of very compact designs which are required to have a low cutoff frequency. Fortunately it is just this requirement which is easiest to realize with the notch frequency well below cutoff. In this regard, the passive-radiator system may be considered as a most natural and logical extension of the vented-box system [10].

9. DISCUSSION

Comparison of Passive-Radiator and Vented-Box Systems

Many of the major differences between vented-box and passive-radiator systems have already been presented in Part I. However, some of the particular similarities and differences merit further discussion.

Driver Requirements

For a given specification of enclosure size, system response, and power capacity, the required driver parameters are virtually the same for both vented-box and passive-radiator systems. Expressed in another way, a particular driver will give substantially the same performance in a given enclosure, regardless of whether the enclosure has a vent or a passive radiator, so long as the passive-radiator compliance ratio δ is high, the passive-radiator losses are not excessively large, and the enclosure is tuned to the correct frequency in each case.

Design Complexity

The additional design complexity of the passive-radiator system is entirely associated with the passive-radiator suspension compliance. Fortunately, this compliance is not critical in the sense that it must always be adjusted to a precise value. The general requirements are easily summed up: allow for the required displacement, and provide maximum compliance (at least equal to that of the driver) with minimum losses. If these requirements are observed, the design of passive-radiator systems is no more complex than that of vented-box systems. The only practical difference is that the required value of f_B is obtained by adjusting the passive-radiator diaphragm mass instead of the acoustic mass of a vent.

* An abridged version of this paper was presented September 10, 1973, at the 46th Convention of the Audio Engineering Society.

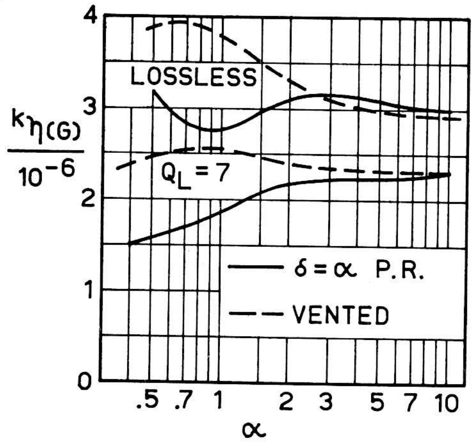


Fig. 13. Response factor $k_{\eta(G)}$ of efficiency constant for $\delta = \alpha$ passive-radiator systems (solid lines) and vented-box systems (broken lines) with lossless enclosures and with $Q_B = Q_L = 7$.

Small-Signal Performance

Fig. 13 (repeated from Part I) shows that the two systems have comparable small-signal performance limits when δ is large. For small values of δ, however, passive-radiator systems have significantly lower values of $k_{\eta(G)}$ than do their vented-box counterparts. This is why passive-radiator suspension compliance should always be made as high as practicable.

For a range of alignments near and above $\alpha = 3$, Fig. 13 shows that the lossless $\delta = \alpha$ passive-radiator system has a value of $k_{\eta(G)}$ slightly greater than that for the lossless vented-box system. Fig. 18 compares the responses for $\alpha = 3$; the driver parameters are virtually identical for both systems. The value of f_3 for the passive-radiator system is indeed about 1% lower, while the cutoff slope is visibly steeper.

For systems with realistic losses, the passive-radiator system appears to be at a disadvantage compared to the vented-box system, although the difference is very small when δ is large. Fig. 19 shows the frequency response measured by the method of [11] for a laboratory driver and test enclosure, first with a vent and again with a passive radiator. The compliance ratios (δ and α) of about unity for this particular system theoretically should favor the use of a vent. The penalty for low passive-radiator compliance is readily apparent in the higher cutoff frequency and steeper initial cutoff slope for the passive radiator.

It is emphasized that the condition $\delta = \alpha$, though common in practice, is used in this paper only as a matter of convenience to simplify the vast range of possible alignments. For best performance it is clearly advisable to use the highest practicable value of passive-radiator compliance.

Large-Signal Performance

Given adequate passive-radiator displacement volume, only small differences are likely to exist in the power capacities of the two systems. These would depend upon the specific relationship between the power spectrum of the driving signal and the exact alignment of the systems.

Popular Beliefs about Passive Radiators

Two particular advantages which are widely claimed for passive-radiator systems, either in popular magazine articles or in advertisements, deserve specific comment in the light of the preceding analysis and discussion.

The first claimed advantage is that the uniform air-particle velocity in the region of the passive radiator is an improvement over the comparatively nonuniform amplitude and phase conditions existing over the aperture of a vent.

This observation first appeared [2, p. 225] in support of a claim that the nonuniform particle velocity in a vent gives rise to vent losses which are eliminated by the use of a passive radiator. This is of course nominally true, but if a vent is properly designed and unobstructed, then the amount of energy dissipated as a result of nonuniform air velocity is relatively small compared to other enclosure losses [5, sec. 3] and easily may be exceeded by that dissipated in the suspension of typical contemporary passive radiators.

Other authors have sometimes misinterpreted the text of [2] and have claimed or suggested that nonuniform particle velocity in a vent is by its very nature inefficient or even nonlinear. But from [4], the relative amplitudes and phases of individual particles are not important. It is their total integrated effect, i.e., the total (phasor sum) volume velocity crossing the enclosure boundaries, that determines the system output. So long as the average particle velocity in the vent is held within the limit discussed in [5, sec 8], all air movement can remain substantially linear and no loss of output or significant nonlinear distortion will occur.

The second claimed advantage of passive-radiator systems (which is particularly popular with advertising copy-

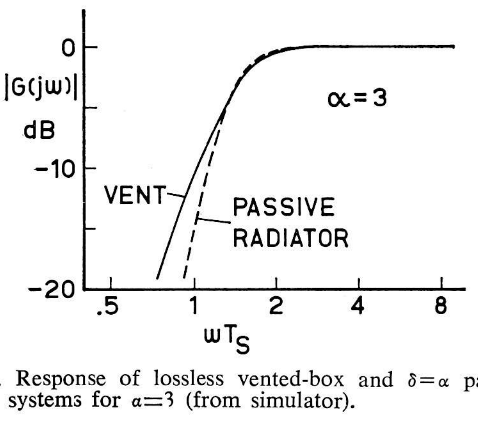


Fig. 18. Response of lossless vented-box and $\delta = \alpha$ passive-radiator systems for $\alpha = 3$ (from simulator).

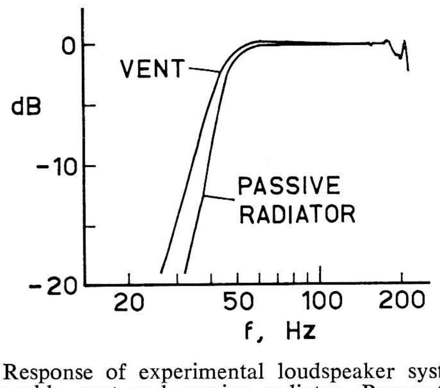


Fig. 19. Response of experimental loudspeaker system with interchangeable vent and passive radiator. Parameters with vent: $\alpha = 1.0$, $h = 1.1$, $Q_T = 0.37$, $Q_B = 9$; passive-radiator compliance ratio $\delta = 1.0$.

writers) is that the use of a passive radiator "doubles the radiating area at low frequencies." It is naturally implied that this is somehow beneficial to performance.

The passive-radiator system does indeed possess the same advantages over the single-diaphragm closed-box system as does the vented-box system [5, Part II]. These advantages, however, depend simply on the *presence* of the secondary aperture, not on its *area*. The passive radiator aids the driver only to the same degree as does a vent. In fact, over the frequency range near f_B where the passive radiator (or vent) contributes most usefully to the system output, it does so through reducing and replacing, rather than supplementing (as so often implied) the motion of the driver.

Additional Features of Passive-Radiator Systems

It might appear from the discussion so far that there is no advantage to using a passive radiator in larger systems for which a satisfactory vent could be realized. Certainly the passive radiator represents a moderate additional cost. Measurements made on systems of this type using interchangeable vents and passive radiators indicate consistently that a passive radiator has greater losses and gives a slightly higher f_3 compared with a vent. But there are at least two features of the passive radiator which do not appear in the basic analysis of the system that are worth taking note of.

First, a passive radiator is entirely free of the windage and resonant-tube noises which are often generated by a vent operated at high volume velocity. So long as the passive radiator is designed to accommodate large linear volume displacements, the total spurious distortion of the passive radiator may then be less.

Second, the passive radiator acts as a physical barrier to the propagation of sound at high frequencies from within the enclosure. Some of the sound coloration which results from the coupling of internal standing-wave modes of the enclosure to the room via natural propagation through the air of a vent is thus substantially reduced or eliminated by the use of a passive radiator.

These two features of the passive-radiator system are perhaps secondary in nature, but they could be important in particular applications.

Typical Passive-Radiator System Performance

During 1969 and 1970 a sample of commercially produced passive-radiator systems was tested by measuring the basic system parameters and obtaining the system response from an analog simulator adjusted to duplicate the system parameters. Only five such systems could be obtained at the time, ranging in enclosure volume from 12 to 56 dm^3 (0.4 to 2 ft^3). They were produced by one manufacturer in the United States and one in Great Britain. Three used 8-in (20-cm) drivers and passive radiators, one used 10-in (25-cm) units, and the last used 12-in (30-cm) units.

Four of the systems had cutoff frequencies f_3 below 50 Hz (the lowest was 39 Hz) and response peaks less than 1 dB. The fifth (and smallest) system had a cutoff frequency of 60 Hz and a response peak of 3 dB; this performance was expected because the enclosure volume was only 12 dm^3 (0.4 ft^3) and the driver and passive radiator were identical to those used in one of the larger systems for which they were more ideally suited.

All systems had values of α and δ equal to or greater than 3, and for the most part these were equal. Three systems had measured Q_B values of 5; the others had values of 4 and 6. Reference efficiencies were all between 0.4 and 0.6%.

All the systems tested were extremely well made and appeared to be the result of very careful testing, as would be expected from these particular manufacturers. It appears that the lack of generally available design information for passive-radiator systems has limited their application to only the most competent manufacturers who have the skill and facilities to carry out careful design and evaluation.

10. SYSTEM SYNTHESIS

System-Component Relationships

The design of passive-radiator systems is exactly analogous to that of vented-box systems [5, sec. 10]. The basic small-signal alignment data are obtained from Fig. 11 (repeated from Part I) for the vast majority of systems having $\delta=\alpha$ and a typical (effective) Q_B value of 7. The alignment chart for vented-box systems with $Q_B = 7$ [5, Fig. 11] is also valid for passive-radiator systems with infinite δ and is reproduced here as Fig. 20. This chart may be used in conjunction with Fig. 11 to interpolate

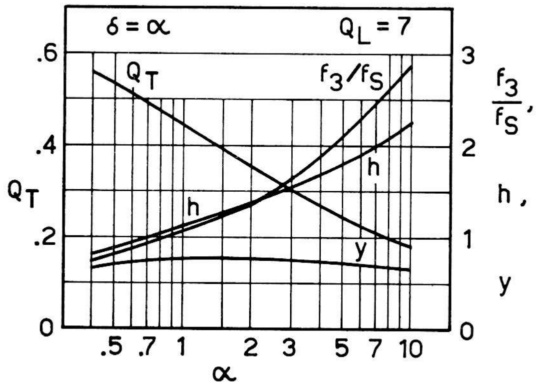


Fig. 11. Alignment chart for $\delta=\alpha$ passive-radiator system with $Q_B=Q_L=7$.

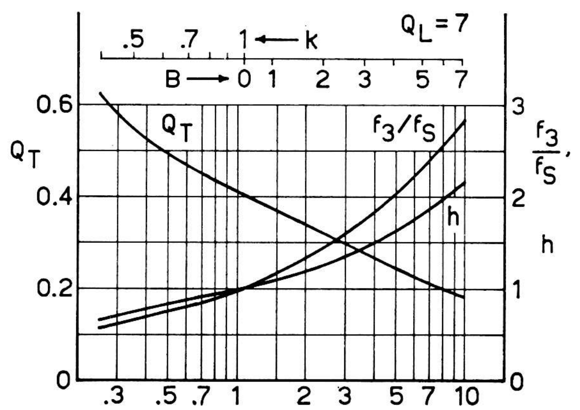


Fig. 20. Alignment chart for vented-box systems with $Q_B=Q_L=7$. Also valid for passive-radiator systems with infinite δ ($y = 0$).

alignments for systems with values of δ greater than α. Comparison of the two figures shows that there is little difference in Q_T or f_3/f_S for large values of α; only h varies noticeably with δ, but not very much.

For unusual design conditions wherein Q_B is quite high or low, but provided that α is large and δ is equal to or greater than α, any of the alignment charts of [5] may be used in place of Fig 11. It is the rarity of either extreme-loss condition, the usefulness of these alternate charts, and the relative unimportance of the actual value of δ (so long as it is large) that make it unnecessary for any charts other than Figs. 11 and 20 to be provided here. For extremely unusual design cases, alignment data may be calculated from the relationships given in the Appendix.

System design procedures are summarized below for both the optimum use of a given driver and the design of a complete system from specifications. Each summary is followed by a specific design example.

Design with a Given Driver

The design of an enclosure and passive radiator to suit a given driver begins with knowledge of the basic small-signal parameters of the driver: f_S, Q_{TS} and V_{AS}. If these are not already known, they may be measured by the method given in [4]. The measurements should be made with the driver on a standard test baffle or the results otherwise adjusted to correspond to the air-mass loading conditions of an enclosure; i.e., it is f_{SB} (and the corresponding value of Q_{TS}), not f_{SA} (the value for free-air loading) that is needed.

The value of Q_{TS} must be no larger than about 0.5 for use in a passive-radiator system. Larger values lead to alignments with excessive passband ripple. It is assumed here that the system will be used with an amplifier having negligible output (Thevenin) resistance so that $Q_T = Q_{TS}$. Thus if the value of Q_{TS} is reasonable, find this value on the Q_T curve in Fig. 11. The value of α on the abscissa corresponding to this value of Q_T is the system compliance ratio required for an optimum "flat" alignment. Using this value of α, the other curves of the figure give the required values of h or y (and therefore f_B or f_P) and the resulting value of f_3 for the system. The required enclosure volume is $V_B = V_{AS}/\alpha$.

The system reference efficiency η_0 is calculated from the driver parameters using Eq. (28). The approximate displacement-limited acoustic power capacity P_{AR} is calculated from Eq. (41) if V_D is known; V_D can be evaluated as described in [8, sec. 6]. The approximate displacement-limited input power capacity P_{ER} is found by dividing P_{AR} by η_0 as indicated by Eq. (42).

If the passive radiator is made from the same frame and suspension as the driver (assuming adequate displacement capability), the diaphragm mass is adjusted to obtain the required value of f_B as indicated by the system impedance curve (see Section 8, Part I).

Example of Design with a Given Driver

It is instructive to repeat here the design example carried out in [5, sec. 10] for two reasons. First, in that example the required vent dimensions of 65-mm (2.6-in) diameter and 175-mm (7-in) length are not wholly desirable. The length is somewhat excessive for a compact enclosure, and the ratio of length to diameter is great

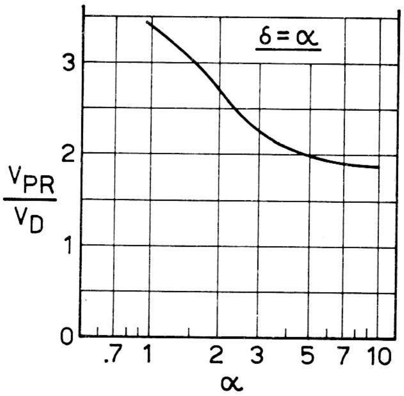


Fig. 16. Required ratio of passive-radiator displacement volume V_{PR} to driver displacement volume V_D as a function of α for program-rated, $\delta = \alpha$ passive-radiator systems (from simulator data).

enough to promote resonant-pipe amplification of vent windage noises. This suggests that a passive radiator would probably give better overall system performance.

Second, the driver parameters used in this example are in fact those of a driver of the same type as that contained in one of the commercial passive-radiator systems described in the previous section. The calculated enclosure design may thus be compared to that found desirable by the manufacturer.

The driver parameters are

$$
\begin{aligned}
f_S &= 33 \text{ Hz}\\
Q_{MS} &= 2.0\\
Q_{ES} &= 0.45\\
V_{AS} &= 57 \text{ dm}^3 \text{ (2 ft}^3\text{)}\\
V_D &= 120 \text{ cm}^3\\
P_{ER} &= \text{(adequate for use with 25-W amplifier)}
\end{aligned}
$$

and by calculation using Eq. (33) and (28),

$$
\begin{aligned}
Q_{TS} &= 0.37\\
\eta_0 &= 0.44\%.
\end{aligned}
$$

For the vented-box design example, the modest enclosure size led to the assumption of $Q_B = 10$. Clearly, the enclosure loss must be higher with the passive radiator, especially if the latter is constructed from the same suspension that produced $Q_{MS} = 2$ for the driver. Hence, using the alignment data from Fig. 11, and assuming negligible driving source impedance so that $Q_T = Q_{TS} = 0.37$, the appropriate system small-signal parameters are

$$
\begin{aligned}
\alpha &= 1.72\\
h &= 1.30 \; (y = 0.79)\\
f_3/f_S &= 1.28
\end{aligned}
$$

and the system design is thus

$$
\begin{aligned}
V_B &= 33 \text{ dm}^3 \text{ (1.2 ft}^3\text{)}\\
f_B &= 43 \text{ Hz } (f_P = 26 \text{ Hz})\\
f_3 &= 42 \text{ Hz.}
\end{aligned}
$$

From Eqs. (41) and (42),

$$
\begin{aligned}
P_{AR} &= 3 f_3^4 V_D^2 = 130 \text{ mW}\\
P_{ER} &= P_{AR}/\eta_0 = 30 \text{ W.}
\end{aligned}
$$

For the 25-W input limit recommend by the manufacturer for this driver, the useful value of P_{AR} is 110 mW.

For $\delta = \alpha$, Fig. 16 suggests that V_{PR} must be about 2.9 times V_D. Because the input power is restricted to 25 W, not quite all of the available V_D is used; the required value of V_{PR} is therefore about 320 cm^3. If the passive radiator has the same diaphragm area as the driver, its total "throw" must be a substantial 32 mm (1.3 in).

The vented-box system designed around this driver in [5, sec. 10] has a 37-dm^3 (1.3-ft^3) enclosure, a cutoff frequency of 38 Hz, but a power capacity of only 90 mW acoustical and 20 W electrical. The passive-radiator design, as a result of its higher cutoff frequency, makes better use of the maximum thermal power capacity of the driver. But because the values of α and especially δ are not particularly high, the value of k_η for this system is noticeably poorer. A higher value of δ (greater passive-radiator suspension compliance), if physically realizable, would be an advantage to this system.

For comparison, the commercial system which uses this driver has the measured properties

$$V_B = 21 \text{ dm}^3 \ (0.74 \text{ ft}^3)$$
$$f_B = 44 \text{ Hz} \ (f_P = 23 \text{ Hz})$$
$$f_3 = 46 \text{ Hz}$$
$$Q_B = 5.1.$$

This represents a higher α (and δ) alignment which has a slight (1-dB) response peak and quite satisfactory cutoff frequency. And significantly, the displacement requirements for both driver and passive radiator are considerably reduced for this system if the input power is still restricted to 25 W.

Design from Specifications

The procedure for designing a passive-radiator system from specifications essentially follows that of [5, sec. 10] for vented-box systems. For passive-radiator systems, however, the range of alignments specified should be limited to system compliance ratios (or at least δ values) of 3 or more. For $\delta = \alpha$ designs, Fig. 11 of the present paper can be used for determination of the driver and passive-radiator small-signal parameters. As with the vented-box system, an alignment with passband peaking may be obtained by allowing a modest increase in Q_T and/or h over the values required for flat response.

The mechanical properties of both driver and passive radiator are calculated from the acoustical requirements by the method of [8, sec. 10] or [5, sec. 11]. The required value of V_{PR} is found from Fig. 16 after the required value of V_D has been calculated.

Example of Design from Specifications

One of the ideal applications of the passive-radiator principle is in compact systems where a low cutoff frequency is required together with a relatively high value of the efficiency constant k_η. Such loudspeaker systems can be expected to provide satisfactory acoustical performance when driven from amplifiers of moderate power and indeed would typically be sold in pairs for use in small rooms together with a stereo amplifier having a continuous power rating of about 15 W per channel. Accordingly, let the system specifications start with the following:

$$V_B = 25 \text{ dm}^3 \ (0.9 \text{ ft}^3)$$
$$f_3 = 40 \text{ Hz}$$
$$P_{ER} = 15 \text{ W}$$

Use: normal program material with 10-dB peak-average power ratio.

The actual alignment has not yet been specified.

For the specified enclosure size it is assumed that both driver and passive radiator must be 8-in (20-cm) units. With such a configuration, it should readily be possible to obtain α and δ values of 3. From Figs. 6, 12, 13, and 16 this alignment provides satisfactory response with a reasonable value of $k_{\eta(G)}$ and a moderate passive-radiator–driver displacement ratio. This completes the system specifications. It is assumed that amplifier driving impedance will be negligible and that system losses will be of normal magnitude.

Design then begins with Fig. 11. For $\delta = \alpha = 3$, the required alignment parameters are

$$Q_T = 0.30$$
$$h = 1.52 \ (y = 0.76)$$
$$f_3/f_S = 1.63.$$

Thus the required driver parameters are

$$f_S = 24.5 \text{ Hz}$$
$$V_{AS} = 75 \text{ dm}^3$$
$$Q_{TS} = 0.30$$

and the passive radiator mass must be adjusted so that

$$f_B = 37.3 \text{ Hz}$$

or, from Eq. (21),

$$f_P = 18.6 \text{ Hz}.$$

If it is assumed that the driver Q_{MS} will be about 3, a typical value for such a driver, then the required electrical damping is

$$Q_{ES} = 0.33.$$

Then from Eq. (28),

$$\eta_o = 0.32\%.$$

From the large-signal specification, Eqs. (42) and (41) give

$$P_{AR} = 15(0.0032) = 48 \text{ mW}$$

and

$$V_D = 80 \text{ cm}^3.$$

From Fig. 16, $V_{PR}/V_D = 2.25$, so

$$V_{PR} = 180 \text{ cm}^3.$$

For 8-in (20-cm) units with a typical diaphragm area of 2.0×10^{-2} m^2, the total "throw" must then be 8 mm (0.31 in) for the driver and 18 mm (0.7 in) for the passive radiator.

Finally, the driver voice coil must be able to dissipate as heat an average nominal input power of at least 1.5 W.

The remaining physical properties of the driver and passive radiator are calculated as outlined in [8, sec. 10]. For the driver, these are

$$C_{MS} = V_{AS}/(\rho_o c^2 S_D^2) = 1.34 \times 10^{-3} \text{ m/N}$$
$$M_{MS} = (\omega_S^2 C_{MS})^{-1} = 31.5 \text{ g}$$
$$M_{MD} = M_{MS} - (\text{air load}) = 28.7 \text{ g (a heavy cone)}$$
$$B^2l^2/R_E = \omega_S M_{MS}/Q_{ES} = 14.7 \text{ N·s/m}$$

or, for $R_E = 6.5\ \Omega$ (typical for 8-Ω rating),

$$Bl = 9.8 \text{ T·m}.$$

Similarly, for the passive radiator,

$$C_{MP} = 1.34 \times 10^{-3} \text{ m/N}$$
$$M_{MP} = 54.3 \text{ g (including air load)}$$
$$M_{MD} = 51.5 \text{ g.}$$

11. CONCLUSION

The passive-radiator loudspeaker system is a nearly equivalent alternative to the vented-box system. It is particularly adaptable to compact enclosures for which a vented-box design cannot be satisfactorily realized.

It is important that the passive-radiator suspension compliance be made as high as conveniently possible and that the displacement limit be large enough to complement the full output capability of the driver. Beyond this, the design requirements are no more difficult than for the vented-box system; maximum performance generally results from the intelligent selection of alignment type and the avoidance of unnecessary losses.

APPENDIX

ELLIPTIC FILTER FUNCTIONS AND ALIGNMENT OF THE LOSSLESS PASSIVE-RADIATOR SYSTEM

General Expressions

The general form of filter function given in Eq. (27) is expressed in magnitude-squared form as

$$|G_H(j\omega)|^2 = \frac{\omega^8T_0^8 + B_1\omega^6T_0^6 + B_2\omega^4T_0^4}{\omega^8T_0^8 + A_1\omega^6T_0^6 + A_2\omega^4T_0^4 + A_3\omega^2T_0^2 + 1} \quad \text{(A-1)}$$

where

$$\begin{aligned} A_1 &= a_1^2 - 2a_2 \\ A_2 &= a_2^2 + 2 - 2a_1 a_3 \\ A_3 &= a_3^2 - 2a_2 \\ B_1 &= -2b_2 \\ B_2 &= b_2^2. \end{aligned} \quad \text{(A-2)}$$

It is convenient to use a restricted form of Eq. (A-1) in which the polynomial coefficients are replaced by constants which relate directly to the types of responses found to be useful. This is

$$|G_H(j\omega)|^2 = \frac{\omega^4T_0^4(k_1^2 - \omega^2T_0^2)^2}{\omega^4T_0^4(k_1^2 - \omega^2T_0^2)^2 + (1 - k_2^2\omega^2T_0^2)^2 + k_3^2\omega^2T_0^2}. \quad \text{(A-3)}$$

It is obvious from Eq. (A-3) that the system response null occurs when $\omega T_0 = k_1$; i.e., k_1 is the normalized frequency of the response null and is equal to $(b_2)^{1/2}$.

For equivalence of Eqs. (A-1) and (A-3),

$$\begin{aligned} B_1 &= -2k_1^2 \\ B_2 &= k_1^4 \\ A_1 &= -2k_1^2 \\ A_2 &= k_1^4 + k_2^4 \\ A_3 &= k_3^2 - 2k_2^2. \end{aligned} \quad \text{(A-4)}$$

This imposes the constraint $A_1 = B_1$, but this constraint is common to all of the responses found useful.

The half-power (− 3 dB) frequency f_3 of any alignment is given by

$$f_3/f_0 = d^{1/2} \quad \text{(A-5)}$$

where

$$f_0 = 1/(2\pi T_0) \quad \text{(A-6)}$$

and d is the largest positive real root of the equation

$$d^4 - (A_1 - 2B_1)d^3 - (A_2 - 2B_2)d^2 - A_3d - 1 = 0. \quad \text{(A-7)}$$

For a response function specified in terms of the values of k_1, k_2, and k_3, the A_i and B_i coefficients can be calculated from Eq. (A-4). Then, using Eq. (A-2), the a_i and b_i coefficients may be found as follows:

$$b_2 = B_2^{1/2} \quad \text{(A-8)}$$

a_2 is found as a positive real root of

$$a_2^4 - 2(A_2 + 6)a_2^2 - 8(A_1 + A_3)a_2 + (A_2 - 2)^2 - 4A_1 A_3 = 0 \quad \text{(A-9)}$$

then

$$\begin{aligned} a_1 &= (A_1 + 2a_2)^{1/2} \\ a_3 &= (A_3 + 2a_2)^{1/2}. \end{aligned} \quad \text{(A-10)}$$

Types of Response

Elliptical Responses [7]

This family of responses is characterized by $k_3 = 0$. The amplitude response has equal-ripple characteristics in both passband and stopband.

Symmetrical Elliptical Responses

This family of responses is characterized by $k_3 = 0$ and $k_2 = k_1$. It has the same properties as the general elliptical family with the addition of the symmetry characteristic

$$G(sT_0) = 1 - G(1/sT_0). \quad \text{(A-11)}$$

Maximally-Flat-Passband-Amplitude Responses [12]

The general maximally flat passband requirement [7] is satisfied by Eq. (A-3) for $k_2 = k_3 = 0$. This requires that, for any suitable value of a_1,

$$\begin{aligned} a_3 &= 2/a_1 + a_1/\sqrt{2} \\ a_2 &= a_3^2/2 \\ b_2 &= a_2 - a_1^2/2. \end{aligned} \quad \text{(A-12)}$$

"Quasi-Maximally-Flat" Responses

The condition of Thiele's "quasi-Butterworth" responses [13] is met by Eq. (A-3) for $k_2 = 0$ and $k_3 > 0$.

Alignment of Lossless Passive-Radiator System

For the lossless passive-radiator system, the response function given in Eq. (22) is equivalent to Eq. (27) for

$$\begin{aligned} T_0 &= (T_PT_S)^{1/2}/\gamma^{1/4} \\ b_2 &= y/\gamma^{1/2} \\ a_1 &= 1/(Q_Ty^{1/2}\gamma^{1/4}) \\ a_2 &= (1/\gamma^{1/2})[y(\delta + 1) + (\alpha + 1)/y] \\ a_3 &= y^{1/2}(\delta + 1)/(Q_T\gamma^{3/4}) \end{aligned} \quad \text{(A-13)}$$

where y is given by Eq. (19) and

$$\gamma = \alpha + \delta + 1. \quad \text{(A-14)}$$

For any given response, the parameters of a lossless system which will produce this response are thus

$$\alpha = \frac{a_2 a_3/a_1 - (a_3/a_1)^2 - 1}{1 - b_2(a_2 - a_3/a_1)}$$
$$\delta = (1/b_2)(a_3/a_1) - 1$$
$$Q_T = 1/[a_1(\gamma b_2)^{1/2}] \qquad \text{(A-15)}$$
$$f_0/f_S = (\gamma b_2)^{1/2}$$
$$h = (\gamma b_2)^{1/2}(a_3/a_1)^{1/2}$$
$$(\text{or } y = \gamma^{1/2} b_2).$$

The normalized cutoff frequency is found from

$$f_3/f_S = (f_0/f_S)(f_3/f_0) = (\gamma b_2)^{1/2}(f_3/f_0). \qquad \text{(A-16)}$$

For the elliptical and quasi-maximally-flat alignments there is an extra degree of freedom, and it is useful to fix an additional parameter relationship so that only a single family of parameter adjustments remains. The practical (and common) restriction $\delta = \alpha$ is used in this paper. This constrains the polynomial coefficients so that

$$\frac{a_3}{a_1} = \frac{2a_2 + b_2 - 1/b_2 \pm \sqrt{(2a_2 + b_2 - 1/b_2)^2 - 8a_2 b_2}}{4}. \qquad \text{(A-17)}$$

For this constraint, the elliptical alignment parameters can be obtained as follows. For a given value of α, find the positive real root r of the equation

$$\frac{(\alpha+1)^4(2\alpha+1)^2}{2\alpha^2} r^3 + \frac{(\alpha+1)^3(2\alpha+1)}{\alpha} r^2 + \frac{(\alpha^2 - \alpha - 1)^2}{2\alpha^2} r - 1 = 0. \qquad \text{(A-18)}$$

Then

$$b_2 = r^{1/2}$$
$$a_3/a_1 = b_2(\alpha + 1)$$
$$a_2 = b_2(\alpha+1) + (\alpha+1)/[b_2(2\alpha+1)]$$
$$a_1 = [2(a_2 - b_2)]^{1/2}$$
$$a_3 = a_1 b_2(\alpha + 1). \qquad \text{(A-19)}$$

The remaining alignment parameters are then found from Eqs. (A-15) and (A-16).

From the calculated alignment data used to construct Fig. 5, if was found that a maximally flat $\delta = \alpha$ alignment occurs for $\delta = \alpha \approx 3.01$. By analogy with the vented-box system, only smaller values of $\delta = \alpha$ should be investigated for elliptical responses, and larger values for quasi-maximally-flat responses. The alignment for which $\delta = \alpha = 1 + \sqrt{2}$ is a symmetrical alignment.

The alignment parameters for quasi-maximally-flat responses are obtained as above except that Eq. (A-18) simplifies to

$$r = \frac{1}{2\alpha + 1} \frac{\sqrt{b^2 + 4ac} - b}{2a} \qquad \text{(A-20)}$$

where

$$a = \alpha(3\alpha + 2)$$
$$b = 2\alpha^2$$
$$c = (\alpha + 1)^2.$$

ACKNOWLEDGMENT

The results reported here were obtained in the course of a post-graduate research program carried out at the School of Electrical Engineering of the University of Sydney with financial support from the Australian Commonwealth Department of Education and Science.

I am also indebted to Dr. H. F. Olson for providing early historical information, to J. E. Benson for suggesting corrections and improvements to the original manuscript, and to Prof. J. R. Ashley for suggesting further useful revisions.

REFERENCES

[1] H. F. Olson, "Loud Speaker and Method of Propagating Sound," U.S. Patent 1,988,250, application Feb. 17, 1934; patented Jan. 15, 1935.

[2] H. F. Olson, J. Preston, and E. G. May, "Recent Developments in Direct-Radiator High-Fidelity Loudspeakers," *J. Audio Eng. Soc.*, vol. 2, pp. 219–227 (Oct. 1954).

[3] H. F. Olson, *Acoustical Engineering* (D. Van Nostrand, Princeton, N.J., 1957), pp. 161–162.

[4] R. H. Small, "Direct-Radiator Loudspeaker System Analysis," *IEEE Trans. Audio Electroacoust.*, vol. AU–19, pp. 269–281 (Dec. 1971); republished in *J. Audio Eng. Soc.*, vol. 20, pp. 383–395 (June 1972).

[5] R. H. Small, "Vented-Box Loudspeaker Systems," *J. Audio Eng. Soc.*, vol. 21, pp. 363–372, 438–444, 549–554, 635–639 (June, July/Aug., Sept., and Oct. 1973).

[6] F. Langford-Smith, *Radiotron Designer's Handbook*, 4th ed. (Wireless Press, Sydney, 1953).

[7] L. Weinberg, *Network Analysis and Synthesis*, Chapter 11 (McGraw-Hill, New York, 1962).

[8] R. H. Small, "Closed-Box Loudspeaker Systems," *J. Audio Eng. Soc.*, vol. 20, no. 10, pp. 798-808 (Dec. 1972); vol. 21, no. 1, pp. 11-18 (Jan./Feb. 1973).

[9] Y. Nomura and Z. Kitamura, "An Analysis of Design Conditions for a Phase-Inverter Speaker System with a Drone Cone," *IEEE Trans. Audio and Electroacoustics*, vol. AU-21, no. 5, pp. 397-407 (October 1973).

[10] B. N. Locanthi, "Application of Electric Circuit Analogies to Loudspeaker Design Problems," *IRE Trans. Audio*, PGA-6, pp. 15-36 (March 1952); republished in *J. Audio Eng. Soc.*, vol. 19, no. 9, pp. 778-785 (October 1971).

[11] R. H. Small, "Simplified Loudspeaker Measurements at Low Frequencies," *Proc. IREE Australia*, vol. 32, no. 8, pp. 299-304 (August 1971); republished in *J. Audio Eng. Soc.*, vol. 20, no. 1, pp. 28-33 (Jan./Feb. 1972).

[12] A. Budak and P. Aronhime, "Maximally Flat Low-Pass Filters with Steeper Slopes at Cutoff," *IEEE Trans. Audio and Electroacoustics*, vol. AU-18, no. 1, pp. 63-66 (March 1970).

[13] A. N. Thiele, "Loudspeakers in Vented Boxes," *Proc. IREE Australia*, vol. 22, pp. 487–508 (Aug. 1961); republished in *J. Audio Eng. Soc.*, vol. 19, pp. 382–392, 471–483 (May and June 1971).

Note: Dr. Small's biography appeared in the October 1974 issue.